

# Riddle Me This: Puzzling over Reinforcing Feedback Loops



In this activity, students will attempt to solve a series of riddles to illustrate how quickly conditions can change as a result of reinforcing feedback loops.

## Objectives

After completing this exercise, students will be able to

- describe exponential growth,
- explain how reinforcing feedback loops create exponential growth, and
- assess claims that increased atmospheric carbon dioxide will be good for forests.

## Materials

- Riddle Me This student page

## Introduction

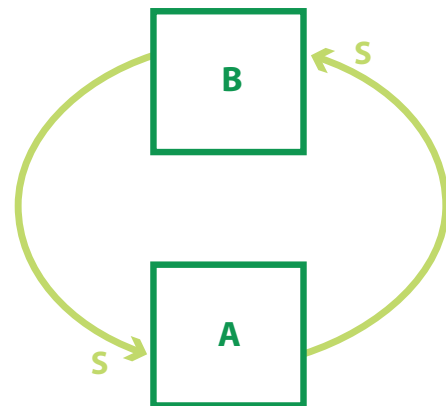
The behavior of complex systems can be surprising at times. By understanding the dynamics of those systems, students can gain a better idea of how they might change over time. In this exercise, students focus on reinforcing feedback loops, which can cause change to happen more rapidly than expected.

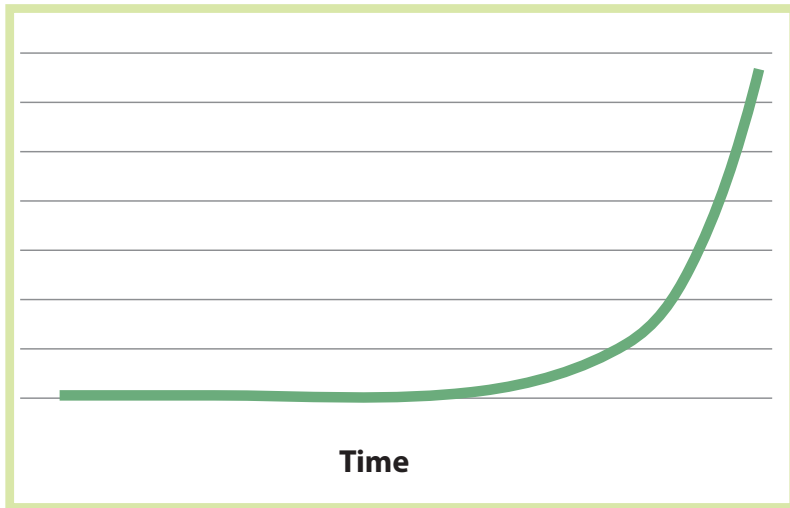
Without an understanding of how these feedback loops affect a system's behavior, students may adopt a more complacent attitude toward climate change, thinking that we can simply start addressing the issue once changes become severe. This exercise also complements Activity 5 by helping students understand feedback in systems.

A reinforcing feedback loop (sometimes called *positive feedback loop*) tends to promote rapid change. The figure to the right illustrates the most basic form of a reinforcing feedback loop, where A and B represent variables and the arrows represent causal connections.

Each arrow is labeled with an S, indicating that the *variables increase or decrease in the same direction*. In other words, the arrow on the left side of the figure indicates that an increase in variable A causes an increase in variable B. The arrow on the right side indicates that an increase in variable B causes an increase in variable A. Therefore, as we move round and round this causal loop, a small change becomes a much larger change.

A reinforcing feedback loop (sometimes called positive feedback loop) tends to promote rapid change.





Reinforcing feedback loops can result in a pattern of exponential growth, as shown here in this J-shaped curve.

The resulting changes often show a pattern of *exponential growth* (or a J-shaped curve) pictured here. Notice that initial changes are gradual, but they become more and more pronounced over time. The student page of this exercise contains three riddles that provide examples of exponential growth. The  $x$ - and  $y$ -axes for each example are determined by the context of the situation, such as 30 days of lily pad growth or 64 pieces on a chessboard.

This exponential pattern of growth is different from linear (or arithmetic) growth. Linear growth indicates that the same quantity is added to the original number over each unit of time. In

exponential growth, the RATE of increase stays the same, but as the number becomes bigger, the increases become bigger. For example, 10% of 100 is 10, while 10% of 1,000 is 100. You can see examples of exponential growth in all kinds of things, from rabbit populations to interest-bearing bank accounts to greenhouse gas emissions. The solutions below show how these dynamics play out in the three riddles. If time permits, you can require students to calculate the correct responses for riddles 2 and 3. The math itself is quite basic and can be done with a calculator (student pages are provided) or an electronic spreadsheet.

Alternatively, instructors can ask students to think through the riddles and then discuss the dynamics at play in each riddle as a class. Conclude by showing students the actual numbers, which are provided in the Answer Key.

### Doing the Exercise

1. Explain the concept of exponential growth and ask students to explore this idea in the three riddles. Distribute the student page. Review responses.
2. Use the riddle solutions in the Answer Key to facilitate a discussion about the students' responses.



# Riddle Me This: Puzzling over Reinforcing Feedback Loops (1 of 3)

NAME \_\_\_\_\_

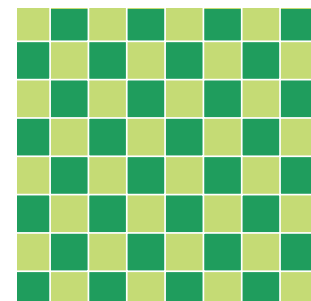
DATE \_\_\_\_\_

Directions: Reinforcing feedback loops can cause systems to change in surprising ways. The following riddles all deal with reinforcing feedback loops. To answer them correctly, you'll need to use your systems thinking skills.

1. A small number of lily pads are placed in a pond on day 1. The lily pads grow and multiply every day so that each day the area of the pond covered by lily pads doubles. On day 30, the pond is completely covered. On what day was the pond half full?
  
2. The thickness of a sheet of notebook paper is approximately 0.1 millimeters. Take a piece of paper and fold it in half. Now it is twice the original thickness:  $0.1 \text{ mm} \times 2 = 0.2 \text{ mm}$ . Now fold it in half again. The thickness doubles again to become 0.4 mm. Approximately how thick would the paper be if you could fold it a total of 50 times?
  - a. About a meter
  - b. 10 meters
  - c. 1 kilometer
  - d. 1000 kilometers
  - e. So thick that it would touch the moon
  
3. The king was in a desperate state. For three years in a row, the kingdom's crops had suffered, and the people were beginning to starve. Then a young man showed up and offered to help the kingdom use new farming techniques. Those techniques worked. The next year's crop was one of the biggest they had ever seen. The king was so appreciative that he offered the young man \$100 million. (Oddly, this kingdom used dollars.)



The young man explained that he did want the \$100 million dollars, but he suggested a different option. He knew the king was a chess player, so he suggested that they put one cent on the first space of the chess board, then two cents on the second space, four cents on the third, and so on, doubling the amount of money on the next space each time. The king could simply pay the young man however much money ended up on the last space (the 64th space). Which payment would be cheaper for the king?





# Riddle Me This: Puzzling over Reinforcing Feedback Loops (2 of 3)

## Riddle 2: Paper Calculation Worksheet

Directions: Fill in the following worksheet to calculate the exact thickness of the paper after each fold.

Fold	Sample Value	Calculated Thickness (km)
<b>Initial Thickness</b>	<b>1</b>	<b>0.0000001</b>
Fold 1	2	
Fold 2	4	
Fold 3	8	
Fold 4	16	
Fold 5	32	
Fold 6	64	
Fold 7	128	
Fold 8	256	
Fold 9		
Fold 10		
Fold 11		
Fold 12		
Fold 13		
Fold 14		
Fold 15		
Fold 16		
Fold 17		
Fold 18		
Fold 19		
Fold 20		
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Fold 22		
Fold 23		
Fold 24		
Fold 25		

Fold	Sample Value	Calculated Thickness (km)
Fold 26		
Fold 27		
Fold 28		
Fold 29		
Fold 30		
Fold 31		
Fold 32		
Fold 33		
Fold 34		
Fold 35		
Fold 36		
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Fold 43		
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Fold 45		
Fold 46		
Fold 47		
Fold 48		
Fold 49		
Fold 50		



# Riddle Me This: Puzzling over Reinforcing Feedback Loops (3 of 3)

## Riddle 3: Chessboard Calculation Worksheet

Directions: Fill in the following worksheet to calculate the exact money value on each chessboard space.

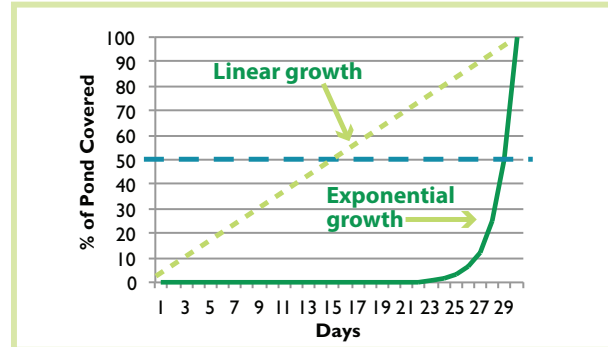
Space	Sample Value	Calculated Value (\$)
<b>Initial Space</b>	<b>1</b>	<b>.01</b>
2	2	
3	4	
4	8	
5	16	
6	32	
7	64	
8	128	
9	256	
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Space	Sample Value	Calculated Value (\$)
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# Riddle Me This: Puzzling over Reinforcing Feedback Loops (1 of 3)

1. Most students will be tempted to guess somewhere around day 15, since that is half of the time. This would be correct if the lilies were increasing linearly, with the same amount of lily pads being added each day. In actuality, the lily pond will be half full on day 29. Then it doubles and becomes completely full on day 30. This graph may help students to observe the difference between linear growth and the exponential growth of the lily pads.

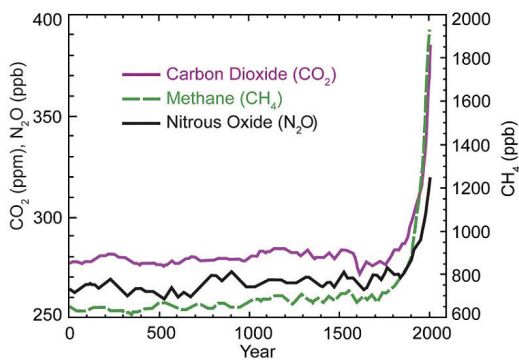


The key lesson to emphasize is how much change occurs in the final days of the lily pad growth. If your goal was to make sure that the pond did not fill up with lily pads (and you did not have systems thinking skills), then you might be complacent about the rate of change of the lily pads all the way through Day 24 or Day 25 (when the lily pads cover less than 5 percent of the pond).

Some individuals adopt similar logic regarding climate change, favoring a wait-and-see approach with the assumption that our communities will have time to make any necessary adjustments only after climate change impacts become more obvious. This approach disregards how quickly reinforcing feedback loops can cause change to happen. There are numerous reinforcing feedback loops that could significantly impact global climate. For example, melting of the polar ice caps could significantly alter the ocean currents which help to regulate climate.

Permafrost (basically, frozen ground) represents another potentially strong feedback loop. Large amounts of carbon are currently stored in permafrost. If the permafrost melts due to increasing temperatures, then much of that carbon could be released into the atmosphere, causing even greater changes in temperature.

## Atmospheric concentrations of greenhouse gases over time



USGCRP, 2009

We need not wait for polar ice caps and permafrost to melt to see reinforcing feedback loops influencing climate change. The EPA's graph illustrated here shows that atmospheric concentrations of greenhouse gases are already increasing exponentially. In this case, both human populations and the amount of energy we use per person are increasing, which create a reinforcing feedback loop for greenhouse gases.

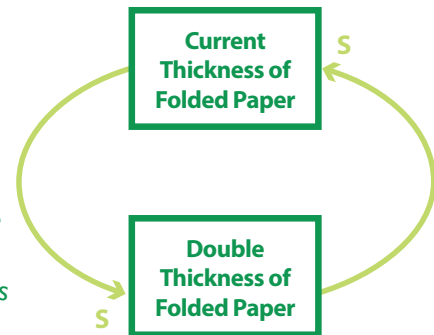
The solutions to the final two riddles provide a clearer sense of how change can escalate over time when reinforcing feedback loops are involved.



# Riddle Me This: Puzzling over Reinforcing Feedback Loops (2 of 3)

2. The thickness of the piece of paper doubles after each fold. In this case, the reinforcing feedback loop looks like the figure to the right. As the paper becomes thicker with each fold, the subsequent fold causes an even greater increase in thickness.

In the table below, the thicknesses are expressed in kilometers ( $0.1 \text{ mm} = 0.0000001 \text{ km}$ ). The table below shows the resulting thickness of each successive fold. Note that after the fiftieth fold, the thickness would be over 112 million kilometers. The distance between the Earth and the moon varies, but 400,000 km is a good estimate. That means that the thickness of the paper would be over 280 times the distance between the earth and the moon. Of course, there is no way to fold a piece of paper that many times—getting past six folds is very difficult—but your students might have fun trying.



A short video illustrating the enormous thickness that would be achieved by 45 folds can be found here: <http://ed.ted.com/lessons/how-folding-paper-can-get-you-to-the-moon>. Note that because they assume thinner paper, the calculated numbers are different. The upshot, however, is the same.

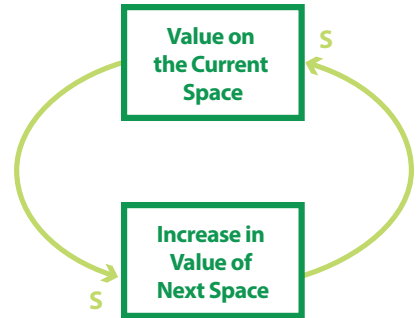
Fold	Thickness (km)	No. of Pages
0	0.0000001	1
1	0.0000002	2
2	0.0000004	4
3	0.0000008	8
4	0.0000016	16
5	0.0000032	32
6	0.0000064	64
7	0.0000128	128
8	0.0000256	256
9	0.0000512	512
10	0.0001024	1,024
11	0.0002	2,048
12	0.0004	4,096
13	0.0008	8,192
14	0.0016	16,384
15	0.0033	32,768
16	0.0066	65,536
17	0.0131	131,072
18	0.0262	262,144
19	0.0524	524,288
20	0.105	$1.05 \times 10^6$
21	0.210	$2.10 \times 10^6$
22	0.419	$4.19 \times 10^6$
23	0.839	$8.39 \times 10^6$
24	1.678	$1.68 \times 10^7$
25	3.355	$3.36 \times 10^7$

Fold	Thickness (km)	No. of Pages
26	6.711	$6.71 \times 10^7$
27	13.422	$1.34 \times 10^8$
28	26.844	$2.68 \times 10^8$
29	53.687	$5.37 \times 10^8$
30	107.374	$1.07 \times 10^9$
31	214.748	$2.15 \times 10^9$
32	429.497	$4.29 \times 10^9$
33	858.993	$8.59 \times 10^9$
34	1,718.0	$1.72 \times 10^{10}$
35	3,436.0	$3.44 \times 10^{10}$
36	6,871.9	$6.87 \times 10^{10}$
37	13,744	$1.37 \times 10^{11}$
38	27,488	$2.75 \times 10^{11}$
39	54,976	$5.50 \times 10^{11}$
40	109,951	$1.10 \times 10^{12}$
41	219,902	$2.20 \times 10^{12}$
42	439,805	$4.40 \times 10^{12}$
43	879,609	$8.80 \times 10^{12}$
44	1,759,219	$1.76 \times 10^{13}$
45	3,518,437	$3.52 \times 10^{13}$
46	7,036,874	$7.04 \times 10^{13}$
47	14,073,749	$1.41 \times 10^{14}$
48	28,147,498	$2.81 \times 10^{14}$
49	56,294,995	$5.63 \times 10^{14}$
50	112,589,991	$1.13 \times 10^{15}$



# Riddle Me This: Puzzling over Reinforcing Feedback Loops (3 of 3)

3. \$100 million is a lot of money. Some of the students might suspect that the young man’s plan is the cheaper way to go. Other students might be suspicious of the plan even without understanding the math. In this case, that suspicion is justified. As in the previous examples, relatively small changes early in the process turn into enormous changes by the time we reach the last space on the chessboard.



The figure **at right** shows the calculated values. The first row shows the value for each space. The other rows show only the value for the last space in each row.

The two graphs **below** illustrate how much larger the value resulting from the chessboard is compared to \$100 million. The graph on the left shows that things start to go sour for the king on the 35th space and get much worse from there. The graph on the right shows that the value of  $9.2 \times 10^{16}$  is so enormous that \$100 million dollars appears to be sitting on the x-axis. Incidentally,  $9.2 \times 10^{16}$  would be 92 quadrillion dollars—over a thousand times larger than the entire global economy today.

Note: This story was adapted from a story in Ibn Kahlilkan’s Biographical Dictionary, published in the 13th Century.

0.01	0.02	0.04	0.08	0.16	0.32	0.64	1.28	
								327.68
								83,886
								21,474,836
								$5.5 \times 10^9$
								$1.4 \times 10^{12}$
								$3.6 \times 10^{14}$
								$9.2 \times 10^{16}$

